
#419547

Topic: Scalars and Vectors

State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Solution

Scalar(only magnitude, no direction): Volume, mass, speed, density, number of moles, angular frequency.

Vector(both magnitude and direction) : Acceleration, velocity, displacement, angular velocity.

#419548

Topic: Scalars and Vectors

Pick out the two scalar quantities in the following list :force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Solution

Work and current are scalar quantities.

(they have no direction)

$$W = \vec{F} \cdot \vec{s}$$

Dot product is scalar.

$$i = V/R$$

Hence scalar.

#419551

Topic: Scalars and Vectors

Pick out the only vector quantity in the following list:

Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Solution

Since, Impulse = change in momentum = force/time.

As momentum and force are vector quantities, hence impulse is a vector quantity.

(vector given by their magnitude and direction)

Impulse is vector, it also has a direction.

#419554

Topic: Scalars and Vectors

State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :

(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions ,(c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Solution

(a) Yes, addition of two scalar quantities is meaningful only if they both represent the same physical quantity.

(b) No, addition of a vector quantity with a scalar quantity is not meaningful.

(c) Yes, scalar can be multiplied with a vector. For example, force is multiplied with time to give impulse.

(d) Yes, scalar, irrespective of the physical quantity it represents, can be multiplied with another scalar having the same or different dimensions.

(e) Yes, addition of two vector quantities is meaningful only if they both represent the same physical quantity.

(f) Yes, component of a vector can be added to the same vector as they both have the same dimensions.

#419557

Topic: Scalars and Vectors

Read each statement below carefully and state with reasons, if it is true or false :

- (a) The magnitude of a vector is always a scalar,
- (b) Each component of a vector is always a scalar,
- (c) The total path length is always equal to the magnitude of the displacement vector of a particle.
- (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time,
- (e) Three vectors not lying in a plane can never add up to give a null vector.

Solution

- (a) True, because magnitude is a pure number and directionless.
- (b) False, vector is sum of its components.(Note: Magnitude of each component is a scalar)
- (c) False, because total path length is greater than or equal to magnitude of displacement vector, whenever direction of motion changes.
- (d) True, because the total path length is either greater than or equal to the magnitude of the displacement vector.
- (e) True, as they can not be represented by the three sides of a triangle taken in the same order.

#419566

Topic: Vector Addition

Establish the following vector inequalities geometrically or otherwise :

- (a) $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$
- (b) $|\mathbf{a} + \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$
- (c) $|\mathbf{a} - \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$
- (d) $|\mathbf{a} - \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$

When does the equality sign above apply?

Solution

(a) Let two vectors $\mathbf{a}$ and $\mathbf{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.

(<http://4.bp.blogspot.com/-jln80SVpNao/VPfbhBKtQzI/AAAAAAAAAETI/6ng3Jf05FEE/s1600/fig-1-chapter-4-class-11th.PNG>) Here, we can write:

$$OM = |\mathbf{a}| \quad \dots(i)$$

$$MN = OP = |\mathbf{b}| \quad \dots(ii)$$

$$ON = |\mathbf{a} + \mathbf{b}| \quad \dots(iii)$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in OMN, we have:

$$ON < (OM + MN)$$

$$|\mathbf{a} + \mathbf{b}| < |\mathbf{a}| + |\mathbf{b}| \quad \dots(iv)$$

If the two vectors $\mathbf{a}$ and $\mathbf{b}$ act along a straight line in the same direction, then we can write:

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}| \quad \dots(v)$$

Combining equations (iv) and (v), we get:

$$|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$

(b) Let two vectors $\mathbf{a}$ and $\mathbf{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.

(<http://2.bp.blogspot.com/-PleYxZC8Rgw/VPfeTd2a6FI/AAAAAAAAAETU/pDanSDJOV94/s1600/fig-2-chapter-4-class-11th.PNG>) Here, we have:

$$OM = |\mathbf{a}|$$

$$MN = OP = |\mathbf{b}|$$

$$ON = |\mathbf{a} + \mathbf{b}|$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in OMN, we have:

$$ON + MN > OM$$

$$ON + OM > MN$$

$$ON > |OM - OP| \quad (OP = MN)$$

$$|\mathbf{a} + \mathbf{b}| > ||\mathbf{a}| - |\mathbf{b}|| \quad \dots(iv)$$

If the two vectors $\mathbf{a}$ and $\mathbf{b}$ act along a straight line in the same direction, then we can write:

$$|\mathbf{a} + \mathbf{b}| = ||\mathbf{a}| - |\mathbf{b}|| \quad \dots(v)$$

Combining equations (iv) and (v), we get:

$$|\mathbf{a} + \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$$

(c) Let two vectors $\mathbf{a}$ and $\mathbf{b}$ be represented by the adjacent sides of a parallelogram PQRS, as shown in the given figure.

(<http://2.bp.blogspot.com/-MquYChZfYRs/VPfeuybb40I/AAAAAAAAETc/hEi564aSit4/s1600/fig-3-chapter-4-class-11th.PNG>)

Here we have:

$$|\mathbf{OR}| = |\mathbf{PS}| = |\mathbf{b}| \quad \dots(i)$$

$$|\mathbf{OP}| = |\mathbf{a}| \quad \dots(ii)$$

In a triangle, each side is smaller than the sum of the other two sides. Therefore, in OPS, we have:

$$|\mathbf{OS}| < |\mathbf{OP}| + |\mathbf{PS}|$$

$$|\mathbf{a} - \mathbf{b}| < |\mathbf{a}| + |\mathbf{b}|$$

$$|\mathbf{a} - \mathbf{b}| < |\mathbf{a}| + |\mathbf{b}| \quad \dots(iii)$$

If the two vectors act in a straight line but in opposite directions, then we can write:

$$|\mathbf{a} - \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}| \quad \dots(iv)$$

Combining equations (iii) and (iv), we get:

$$|\mathbf{a} - \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$

(d) Let two vectors $\mathbf{a}$ and $\mathbf{b}$ be represented by the adjacent sides of a parallelogram PQRS, as shown in the given figure.

(http://4.bp.blogspot.com/-tWspYbA1jaw/VPf8g8c9hRI/AAAAAAAAETo/_qiQdKu1NNA/s1600/fig-4-chapter-3-class-11th.PNG)

The following relations can be written for the given parallelogram. $|\mathbf{OS}| + |\mathbf{PS}| > |\mathbf{OP}| \quad \dots(i)$

$$|\mathbf{OS}| > |\mathbf{OP}| - |\mathbf{PS}| \quad \dots(ii)$$

$$|\mathbf{a} - \mathbf{b}| > |\mathbf{a}| - |\mathbf{b}| \quad \dots(iii)$$

The quantity on the LHS is always positive and that on the RHS can be positive or negative. To make both quantities positive, we take modulus on both sides as:

$$||\mathbf{a} - \mathbf{b}|| > ||\mathbf{a}| - |\mathbf{b}||$$

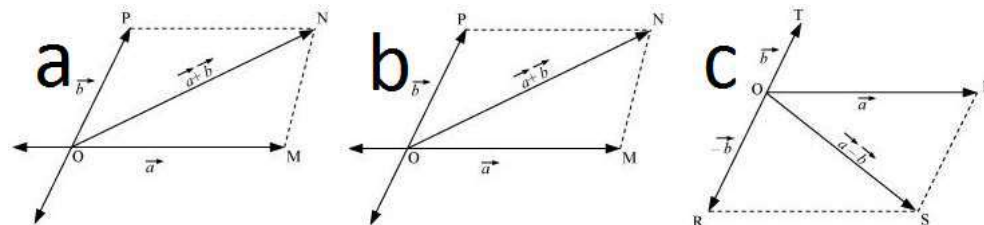
$$|\mathbf{a} - \mathbf{b}| > ||\mathbf{a}| - |\mathbf{b}|| \quad \dots(iv)$$

If the two vectors act in a straight line but in the opposite directions, then we can write:

$$|\mathbf{a} - \mathbf{b}| = ||\mathbf{a}| - |\mathbf{b}|| \quad \dots(v)$$

Combining equations (iv) and (v), we get:

$$|\mathbf{a} - \mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$$



#419574

Topic: Vector Addition

Given $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \mathbf{0}$, which of the following statements are correct.

- (a) $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ must each be a null vector,
- (b) The magnitude of $(\mathbf{a} + \mathbf{c})$ equals the magnitude of $(\mathbf{b} + \mathbf{d})$,
- (c) The magnitude of $\mathbf{a}$ can never be greater than the sum of the magnitudes of $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$,
- (d) $\mathbf{b} + \mathbf{c}$ must lie in the plane of $\mathbf{a}$ and $\mathbf{d}$ if $\mathbf{a}$ and $\mathbf{d}$ are not collinear, and in the line of $\mathbf{a}$ and $\mathbf{d}$, if they are collinear ?

Solution

(a) In order to make vectors $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \mathbf{0}$, it is not necessary to have all the four given vectors to be null vectors. There are many other combinations which can give the sum zero.

(b) Correct

$$\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \mathbf{0}$$

$$\mathbf{a} + \mathbf{c} = -(\mathbf{b} + \mathbf{d})$$

Taking modulus on both the sides, we get:

$$|\mathbf{a} + \mathbf{c}| = |-(\mathbf{b} + \mathbf{d})| = |\mathbf{b} + \mathbf{d}|$$

Hence, the magnitude of $(\mathbf{a} + \mathbf{c})$ is the same as the magnitude of $(\mathbf{b} + \mathbf{d})$.

(c) Correct

$$\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \mathbf{0}$$

$$\mathbf{a} = -(\mathbf{b} + \mathbf{c} + \mathbf{d})$$

Taking modulus both sides, we get:

$$|\mathbf{a}| = |\mathbf{b} + \mathbf{c} + \mathbf{d}|$$

$$|\mathbf{a}| \leq |\mathbf{b}| + |\mathbf{c}| + |\mathbf{d}| \dots (i)$$

Equation (i) shows that the magnitude of $\mathbf{a}$ is equal to or less than the sum of the magnitudes of $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$.

Hence, the magnitude of vector $\mathbf{a}$ can never be greater than the sum of the magnitudes of $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$.

(d) Correct

$$\text{For } \mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \mathbf{0}$$

$$\mathbf{a} + (\mathbf{b} + \mathbf{c}) + \mathbf{d} = \mathbf{0}$$

The resultant sum of the three vectors $\mathbf{a}$, $(\mathbf{b} + \mathbf{c})$, and $\mathbf{d}$ can be zero only if $(\mathbf{b} + \mathbf{c})$ lie in a plane containing $\mathbf{a}$ and $\mathbf{d}$, assuming that these three vectors are represented by the three sides of a triangle.

If $\mathbf{a}$ and $\mathbf{d}$ are collinear, then it implies that the vector $(\mathbf{b} + \mathbf{c})$ is in the line of $\mathbf{a}$ and $\mathbf{d}$. This implication holds, only then the vector sum of all the vectors will be zero.

#419602

Topic: Special Cases of Relative Motion

Rain is falling vertically with a speed of 30 m s^{-1} . A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella?

Solution

The described situation is shown in the given figure.

(http://1.bp.blogspot.com/-hIPt7_bld-s/VPgIK-OwWKI/AAAAAAAEUs/YndNBpjs2Q/s1600/graph-1-chapter-4-class-11th.png) Here, v_c = Velocity of the cyclist

v_r = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity (v) of the rain with respect to the woman.

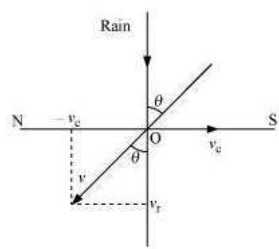
$$v = v_r + (-v_c)$$

$$= 30 + (-10) = 20 \text{ m/s}$$

$$\tan \theta = v_c / v_r = 10/30$$

$$\theta = 18^\circ$$

Hence, the woman must hold the umbrella toward the south, at an angle of nearly 18° with the vertical.



#419612

Topic: Basics of Projectile Motion

A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

Solution

Maximum horizontal distance, $R = 100\text{m}$

The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is 45° , i.e., $\theta = 45^\circ$.

The max horizontal range for a projection velocity u , is given by the relation:

$$R_{\max} = u^2 / g$$

$100 = u^2 / g$ The ball will achieve the maximum height when it is thrown vertically upward. For such motion, the final velocity v is zero at the maximum height H .

Acceleration, $a = g$

Using the third equation of motion:

$$v^2 - u^2 = -2gH$$

$$H = u^2 / 2g = 100/2 = 50\text{m}$$

#419623

Topic: Relative Motion

A truck starts from rest and accelerates uniformly at 2.0 m s^{-2} . At $t = 10 \text{ s}$, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity, and (b) acceleration of the stone at $t = 11 \text{ s}$? (Neglect air resistance.)

Solution

(a) Initial velocity of the truck, $u = 0$

Acceleration, $a = 2 \text{ m/s}^2$

Time, $t = 10 \text{ s}$

As per the first equation of motion, final velocity is given as:

$$v = u + at$$

$$= 0 + 2 \times 10 = 20 \text{ m/s}$$

The final velocity of the truck and hence, of the stone is 20 m/s .

At $t = 11 \text{ s}$, the horizontal component (v_x) of velocity, in the absence of air resistance, remains unchanged, i.e.,

$$v_x = 20 \text{ m/s}$$

The vertical component (v_y) of velocity of the stone is given by the first equation of motion as:

$$v_y = u + at$$

Where, $t = 11 - 10 = 1 \text{ s}$ and $a_y = g = 10 \text{ m/s}^2$

$$v_y = 0 + 10 \times 1 = 10 \text{ m/s}$$

The resultant velocity (v) of the stone is given as:

(<http://1.bp.blogspot.com/-F9ICSJdvGrS/VPre4-T6g2I/AAAAAAAAEYg/tZ4rZgw8f9c/s1600/fig-2-chapter-5-class-11th.png>)

$$v = (v_x^2 + v_y^2)^{1/2}$$

$$= (20^2 + 10^2)^{1/2}$$

$$= 22.36 \text{ m/s}$$

Let θ be the angle made by the resultant velocity with the horizontal component of velocity, v_x be θ

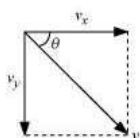
$$\tan \theta = v_y / v_x$$

$$\theta = \tan^{-1} (v_y / v_x)$$

$$= \tan^{-1} (10 / 20)$$

$$= 26.57^\circ$$

(b) When the stone is dropped from the truck, the horizontal force acting on it becomes zero. However, the stone continues to move under the influence of gravity. Hence, the acceleration of the stone is 10 m/s^2 and it acts vertically downward.



#419704

Topic: Scalars and Vectors

Read each statement below carefully and state, with reasons and examples, if it is true or false :

A scalar quantity is one that ;

- (a) is conserved in a process.
- (b) can never take negative values
- (c) must be dimensionless.
- (d) does not vary from one point to another in space.
- (e) has the same value for observers with different orientations of axes.

Solution

- (a) False: Scalars need not be constant. Example: distance travelled by a moving object.
- (b) Scalars can be positive as well as negative, for eg. Temperature can be -20 deg Celsius. Hence, above statement is false.
- (c) Scalar quantities have dimensions, they do not have directions. Distance is scalar and has dimension. Hence, it is a false statement.
- (d) False: A scalar quantity such as gravitational potential can vary from one point to another in space.
- (e) Scalar quantity does not vary with axes, hence the above statement is true.

#419720

Topic: Scalars and Vectors

A vector has magnitude and direction. Does it have a location in space ? Can it vary with time ? Will two equal vectors **a** and **b** at different locations in space necessarily have identical physical effects ? Give examples in support of your answer.

Solution

- (i) A vector in general has no definite location in space because a vector remains unaffected whenever it is displaced anywhere in space provided its magnitude and direction do not change. However, a position vector has a definite location in space.
- (ii) A vector can vary with time as example the velocity vector of an accelerated particle varies with time.
- (iii) Two equal vectors at different locations in space do not necessarily have same physical effects. For example, two equal forces acting at two different points on a body which can cause the rotation of a body about an axis will not produce equal turning effect.

#419721

Topic: Scalars and Vectors

A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector ?

Solution

No. A physical quantity having both magnitude and direction need not be considered a vector. For example, despite having magnitude and direction, current is a scalar quantity. The essential requirement for a physical quantity to be considered a vector is that it should follow the law of vector addition. Generally speaking, the rotation of a body about an axis is not a vector quantity as it does not follow the law of vector addition. However, a rotation by a certain small angle follows the law of vector addition and is therefore considered a vector.

#419722

Topic: Scalars and Vectors

Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere ? Explain.

Solution

- (a) No, one cannot associate a vector with the length of a wire bent into a loop.
- (b) Yes, one can associate an area vector with a plane area. The direction of this vector is normal, inward or outward to the plane area.
- (c) No, one cannot associate a vector with the volume of a sphere. However, an area vector can be associated with the area of a sphere.

#419724

Topic: Basics of Projectile Motion

A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the muzzle speed to be fixed, and neglect air resistance.

SolutionRange, $R = 3$ kmAngle of projection, $= 30^\circ$ Acceleration due to gravity, $g = 9.8 \text{ m/s}^2$ Horizontal range for the projection velocity u_0 , is given by the relation:

$$R = u_0^2 \sin 2\theta / g$$

$$3 = u_0^2 \sin 60^\circ / g$$

$$u_0^2 / g = 2\sqrt{3} \quad \dots\dots(i)$$

The maximum range ($R_{\max}$) is achieved by the bullet when it is fired at an angle of 45° with the horizontal, that is, $R_{\max} = u_0^2 / g \quad \dots(ii)$

On comparing equations (i) and (ii), we get:

$$R_{\max} = 2 \times 1.732 = 3.46 \text{ km}$$

Hence, the bullet will not hit a target 5 km away.

#419726

Topic: Basics of Projectile Motion

A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take $g = 10 \text{ m s}^{-2}$)

SolutionHeight of the fighter plane $= 1.5 \text{ km} = 1500 \text{ m}$ Speed of the fighter plane, $v = 720 \text{ km/h} = 200 \text{ m/s}$ Let θ be the angle with the vertical so that the shell hits the plane. The situation is shown in the given figure.

(<http://2.bp.blogspot.com/-TAOiEnmir3Y/VPqkw-ASNzI/AAAAAAAAAEXo/OzYkgCH5ErY/s1600/fig-7-chapter-4-class-11th.PNG>)

Muzzle velocity of the gun, $u = 600 \text{ m/s}$ Time taken by the shell to hit the plane $= t$ Horizontal distance travelled by the shell $= u_x t$ Distance travelled by the plane $= vt$

The shell hits the plane. Hence, these two distances must be equal.

$$u_x t = vt$$

$$u \sin \theta = v$$

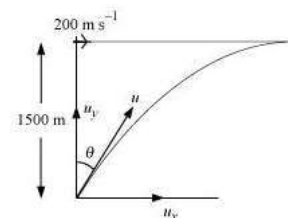
$$\sin \theta = v / u$$

$$= 200 / 600 = 1/3 = 0.33$$

$$= \sin^{-1}(0.33) = 19.50^\circ$$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (H) higher than the maximum height achieved by the shell for any angle of launch.

$$H_{\max} = u^2 / 2g = 600^2 / (2 \times 10) = 18 \text{ km}$$



#419758

Topic: Kinematics of Circular Motion

A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min . The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?

Solution

Mass of the man, $m = 70 \text{ kg}$

Radius of the drum, $r = 3 \text{ m}$

Coefficient of friction, $\mu = 0.15$

Frequency of rotation, $\omega = 200 \text{ rev/min} = 200/60 = 10/3 \text{ rev/s}$

The necessary centripetal force required for the rotation of the man is provided by the normal force (F_N).

When the floor revolves, the man sticks to the wall of the drum. Hence, the weight of the man (mg) acting downward is balanced by the frictional force

($f = F_N$) acting upward.

Hence, the man will not fall until:

$$mg < F_N = \mu m r \omega^2$$

$$\omega > \sqrt{g/r\mu}$$

$$= (10/(0.15 \times 3))^{1/2} = 4.71 \text{ rad/s}$$

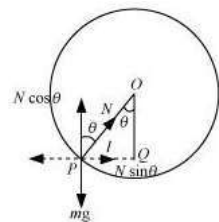
#419761

Topic: Kinematics of Circular Motion

A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω . Show that a small bead on the wire loop remains at its lower most point for $\omega \leq \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

Solution

Let the radius vector joining the bead with the centre make an angle θ , with the vertical downward direction.



(<http://4.bp.blogspot.com/-64gKLBEn9VE/VPwHRkSq2II/AAAAAAAAAEb8/7phkCTKFpOI/s1600/fig-12-chapter-5-class-11th.png>)

OP = R = Radius of the circle

N = Normal reaction

The respective vertical and horizontal equations of forces can be written as:

$$mg = N \cos \theta \quad \text{... (i)}$$

$$m\omega^2 R \sin \theta = N \sin \theta \quad \text{... (ii)}$$

In $\triangle OPQ$, we have:

$$\sin \theta = PQ / R$$

$$PQ = R \sin \theta \quad \text{... (iii)}$$

Substituting equation (iii) in equation (ii), we get:

$$m(R \sin \theta) \omega^2 = N \sin \theta$$

$$mR \omega^2 = N \quad \text{... (iv)}$$

Substituting equation (iv) in equation (i), we get:

$$mg = mR \omega^2 \cos \theta$$

$$\cos \theta = g / R \omega^2 \quad \text{... (v)}$$

Since $\cos \theta \leq 1$, the bead will remain at its lowermost point for $g / R \omega^2 \leq 1$, i.e., for $\omega \leq (g / R)^{1/2}$

$$\text{For } \omega = (2g / R)^{1/2} \text{ or } \omega^2 = 2g / R \quad \text{... (vi)}$$

On equating equations (v) and (vi), we get:

$$2g / R = g / R \cos \theta$$

$$\cos \theta = 1/2$$

$$\therefore \theta = \cos^{-1}(0.5) = 60^\circ$$

#458297

Topic: Vector Addition

Show that the area of the triangle contained between the vectors **a** and **b** is one half of the magnitude of $\mathbf{a} \times \mathbf{b}$.

Solution

Consider two vectors $OK = \text{vector } |\mathbf{a}|$ and $OM = \text{vector } |\mathbf{b}|$, inclined at an angle θ as shown in the following figure.

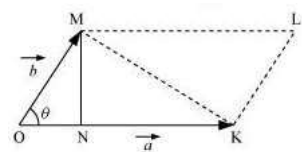
In $\triangle OMN$, we can write the relation:

$$\sin\theta = \frac{MN}{OM} = \frac{MN}{|\mathbf{b}|}$$

$$\implies MN = |\mathbf{b}| \sin\theta$$

$$\begin{aligned} |\vec{a} \times \vec{b}| &= |\vec{a}| |\vec{b}| \sin\theta \\ &= OK \times MN \\ &= 2 \times \frac{1}{2} \times OK \times MN \\ &= 2 \times \text{Area of } \triangle OMK \end{aligned}$$

$$\implies \text{Area of } \triangle OMK = \frac{1}{2} \times |\vec{a} \times \vec{b}|$$



#458299

Topic: Vector Addition

Show that $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors, **a**, **b** and **c**.

Solution

A parallelepiped with origin O and sides $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is shown in the following figure.

Volume of parallelepiped = abc

Let $\hat{n}$ be a unit vector perpendicular to both $\vec{b}$ and $\vec{c}$. Hence $\hat{n}$ and $\vec{c}$ have the same direction.

Therefore, $\vec{b} \times \vec{c} = bc \sin\theta \hat{n} = bc \hat{n}$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = a \cdot (bc \hat{n})$$

$$= abc = \text{Volume of parallelepiped}$$

