Limits and Derivatives Formulas

1. Limits

Properties

if $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$, then

$$\lim_{x\to a}\left[f(x)\pm g(x)\right] = l \pm m$$

$$\lim_{x\to a}\left[f(x)\cdot g(x)\right] = l \cdot m$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{l}{m} \text{ where } m \neq 0$$

$$\lim_{x\to a} c\cdot f(x) = c\cdot l$$

$$\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{l} \text{ where } l \neq 0$$

Formulas

$$\lim_{x\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{x\to\infty}(1+n)^{\frac{1}{n}} = e$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{\tan x}{x} = 1$$

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$

$$\lim_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\lim_{x\to 0}\frac{a^n - 1}{x} = \ln a$$

2. Common Derivatives

Basic Properties and Formulas

$$(cf)' = cf'(x)$$

$$(f \pm g)' = f'(x) + g'(x)$$

Product rule

$$(f\cdot g)' = f'\cdot g + f\cdot g'$$

Quotient rule

$$\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$$

Power rule

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Chain rule

$$\frac{d}{dx}\left(f\left(g\left(x\right)\right)\right) = f'\left(g(x)\right)g'(x)$$

Common Derivatives

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc \cot x$$

$$\frac{d}{dx}(\cot x) = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x},\quad x > 0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x},\ x \neq 0$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a},\ x > 0$$

3. Higher-order Derivatives

Definitions and properties

Second derivative

$$f'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) - \frac{d^2 y}{dx^2}$$

Higher-Order derivative

$$f^{(n)} = \left(f^{(n-1)}\right)'$$

$$(f+g)^{(n)} = f^{(n)} + g^{(n)}$$

$$(f-g)^{(n)} = f^{(n)} - g^{(n)}$$

Leibniz's Formulas

$$(f \cdot g)'' = f'' \cdot g + 2f' \cdot g' + f.g''$$

$$(f \cdot g)''' = f''' \cdot g + 3f'' \cdot g' + 3f' \cdot g'' + f \cdot g'''$$

$$(f \cdot g)^{(n)} = f^{(n)} g + nf^{(n-1)} g + \frac{n(n-1)}{1 \cdot 2} f^{(n-2)} g'' + \ldots + fg^{(n)}$$

Important Formulas

$$\left(x^m\right)^{(n)} = \frac{m!}{(m-n)!} x^{m-n}$$

$$\left(x^n\right)^{(n)} = n!$$

$$(\log_a x)^{(n)} = \frac{(-1)^{n-1}(n-1)!}{x^n \cdot \ln a}$$

$$(\ln x)^{(n)} = \frac{(-1)^{n-1}(n-1)!}{x^n}$$

$$\left(a^x\right)^{(n)} = a^x \ln^n a$$

$$\left(e^x\right)^{(n)} = e^x$$

$$\left(a^{mx}\right)^{(n)} = m^n a^{mx} \ln^n a$$

$$(\sin x)^{(n)} = \sin\left(x + \frac{n\pi}{2}\right)$$

$$(\cos x)^{(n)} = \cos\left(x + \frac{n\pi}{2}\right)$$