#418183

Topic: Introduction

A point is on the x-axis. What are its y-coordinate and z-coordinates?

Solution

If a point is on the x-axis then only x-coordinate will have non-zero constant value and other coordinates will be zero.

Hence, y-coordinates and z-coordinates are zero.

#418185

Topic: Introduction

If a point is in the XZ-plane. What can you say about its y-coordinate?

Solution

If a point is in the XZ plane then its y-coordinate is zero

#418190

Topic: Introduction

Name the octants in which the following points lie:

$(1, 2, 3), (4, -2, 3)(4, -2, -5), (4, 2, -5), (-4, 2, -5), (-4, 2, 5), (-3, -1, 6), (2, -4, -7)$

Solution

The x-coordinate y-coordinate and$z$-coordinate of point $(1, 2, 3)$ are all positive.

Therefore this point lies in octant I

The x-coordinate y-coordinate and z-coordinate of point $(4, -2, 3)$ are positive

negative and positive respectively. Therefore this point lies in octant IV

The x-coordinate y-coordinate and z-coordinate of point $(4, -2, -5)$ are positive

negative and negative respectively. Therefore this point lies in octant VIII

The x-coordinate, y-coordinate and z-coordinate of point $(4, 2, -5)$ are positive

positive and negative respectively. Therefore this point lies in octant V

The x-coordinate, y-coordinate and z-coordinate of point $(-4, 2, -5)$ are negative

positive, and negative respectively. Therefore this point lies in octant VI

The x-coordinate, y-coordinate and z-coordinate of point $(-4, 2, 5)$ are negative

positive and positive respectively. Therefore this point lies in octant II

The x-coordinate y-coordinate and z-coordinate of point $(-3, -1, 6)$ are negative

negative and positive respectively. Therefore this point lies in octant III

The x-coordinate, y-coordinate and z-coordinate of point $(2, -4, -7)$ are positive

negative and negative respectively. Therefore this point lies in octant VIII

#418193

Topic: Introduction

Fill in the blanks:

(i) The x-axis and y-axis taken together determine a plane known as_______

(ii) The coordinates of points in the XY-plane are of the form______

(iii) Coordinate planes divide the space into______ octants

Solution

(i) The x-axis and y-axis taken together determine a plane known as XY-plane

(ii) The coordinates of points in the XY-plane are of the form (x, y), where x is x-coordinate and y is y-coordinate

(iii) Coordinate planes divide the space into 8 octants.

#418205

Topic: Distance Between Two Points

Find the distance between the following pairs of points:

(i) $(2, 3, 5)$ and $(4, 3, 1)$

(ii)$(-3, 7, 2)$and $((2, 4, -1)$

(iii) $(-1, 3, -4)$ and $(1, -3, 4)$

(iv) $(2, -1, 3)$ and $(-2, 1, 3)$

Solution

The distance between points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$PQ = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$

(i) Distance between points $(2, 3, 5)$ and $(4, 3, 1)$

$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

(ii) Distance between points $(-3, 7, 2)$ and $(2, 4, -1)$

$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$

$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$

$= \sqrt{25+9+9}$

$= \sqrt{43}$

(iii) Distance between points $(-1, 3, -4)$ and $(1, -3, 4)$

$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (8)^2}$

$= \sqrt{4+36+64} = \sqrt{104}$

$= 2\sqrt{26}$

(iv) Distance between points $(2, -1, 3)$ and $(-2, 1, 3)$

$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$

$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

$= 2\sqrt{5}$

#418298

Topic: Distance Between Two Points

Verify the following

(i) $(0, 7, -10), (1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle

(ii) $(0, 7, 10), (-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right angled triangle

(iii) $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram

Solution

(i) Let point $(0, 7, -10)$, $(1, 6, -6)$ and $(4, 9, -6)$ be denoted by A, B and C respectively

$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2}$

$= \sqrt{(1)^2 + (-1)^2 + (4)^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$\Rightarrow AB = 3\sqrt{2}$

$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9+9} = \sqrt{18}$

$\Rightarrow BC = 3\sqrt{2}$

$CA = \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2}$

$= \sqrt{(-4)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{16+4+16} = \sqrt{36} = 6$

Here $AB = BC \neq CA$

Thus the given points are the vertices of an isosceles triangle

(ii) Let $(0, 7, 10)$, $(-1, 6, 6)$ and $(-4, 9, 6)$ be denoted by A, B and C respectively

$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + (-4)^2}$

$= \sqrt{1+1+16} = \sqrt{18}$

$= 3\sqrt{2}$

$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$

$= \sqrt{(-3)^2 + (3)^2 + (0)^2}$

$= \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$

$CA = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$

$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

$= 6$

Now $AB^2 + BC^2 = \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 = 18 + 18 = 36 = AC^2$

Therefore by pythagoras theorem ABC is a right triangle

Hence the given points are the vertices of a right-angled triangle

(iii) Let $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$ and $(2, -3, 4)$ be denoted by A, B, C and D respectively

$AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$

$= \sqrt{4+16+16}$

$AB = \sqrt{36}$

$AB = 6$

$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$

$= \sqrt{9+25+9} = \sqrt{43}$

$CD = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$

$= \sqrt{4+16+16}$

$= \sqrt{36}$

$CD = 6$

$DA = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$

$DA = \sqrt{9+25+9} = \sqrt{43}$

Here $AB = CD = 6$, $BC = AD = \sqrt{43}$

Hence the opposite sides of quadrilateral $ABCD$ whose vertices are taken in order are equal

Therefore $ABCD$ is a parallelogram

Hence the given points are the vertices of a parallelogram

#418308

Topic: Distance Between Two Points

Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$

Solution

Let $P(x, y, z)$ be the point that is equidistant from points $A(1, 2, 3)$ and $B(3, 2, -1)$.

i.e. $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 6z + 14 = -6x + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus the required equation is $x - 2z = 0$.

#418320

Topic: Distance Between Two Points

Find the equation of the set of points P, the sum of whose distances from $A(4, 0, 0)$ and $B(-4, 0, 0)$ is equal to 10

Solution

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are $(4, 0, 0)$ and $(-4, 0, 0)$ respectively.

It is given that $PA + PB = 10$

$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$

$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$

On squaring both sides, we obtain

$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 + (x+4)^2 + y^2 + z^2 - 20\sqrt{(x+4)^2 + y^2 + z^2}$

$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$

$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$

$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$

On squaring both sides again, we obtain

$25\left(x^2 + 8x + 16 + y^2 + z^2\right) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

#418328

Topic: Section Formula

Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio

(i) $2:3$ internally

(ii) $2:3$ externally

Solution

(i) The coordinates of point R that divides the line segment joining points $P\left(x_1, y_1, z_1\right)$ and $Q\left(x_2, y_2, z_2\right)$ internally in the ratio $m:n$ are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

Let $R(x, y, z)$ be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ internally in the ratio $2:3$

$x = \frac{2(1)+3(-2)}{2+3}$, $y = \frac{2(-4)+3(3)}{2+3}$ and $z = \frac{2(6)+3(5)}{2+3}$

i.e., $x = \frac{-4}{5}$, $y = \frac{1}{5}$ and $z = \frac{27}{5}$

Thus the coordinates of the required point are $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)$

(ii) The coordinates of point R that divides the line segment joining points $P\left(x_1, y_1, z_1\right)$ and $Q\left(x_2, y_2, z_2\right)$ externally in the ratio $m:n$ are

$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$$

Let $R(x, y, z)$ be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ externally in the ratio $2:3$

$x = \frac{2(1)-3(-2)}{2-3}$, $y = \frac{2(-4)-3(3)}{2-3}$ and $z = \frac{2(6)-3(5)}{2-3}$

i.e.,$x = -8$, $y = 17$ and $z = 3$

Thus the coordinates of the required point are $(-8, 17, 3)$

#418342

Topic: Section Formula

Given that $P(3, 2, -4)$, $Q(5, 4, -6)$ and $R(9, 8, -10)$ are collinear. Find the ratio in which Q divides PR.

Solution

Given points $P(3, 2, -4)$, $Q(5, 4, -6)$ and $R(9, 8, -10)$

Let Q divides PR in the ratio $k:1$.

So, by section formula, coordinates of Q are $\left(\frac{k(9)+1(3)}{k+1}, \frac{k(8)+1(2)}{k+1}, \frac{k(-10)+1(-4)}{k+1}\right.$

$= \left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k+4}{k+1}\right)$

But the given coordinates of Q are $(5, 4, -6)$.

On comparing $\frac{9k+3}{k+1} = 5$

$\Rightarrow 9k+3 = 5k+5$

$\Rightarrow k = \frac{1}{2}$

So, Q divides PQ in the ratio $1:2$.

Let point $Q(5, 4, -6)$ divides the line segment joining points $P(3, 2, -4)$ and $R(9, 8, -10)$ in the ration $k:1$.

Thus using section formula,

$(5, 4, -6) = \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1}\right)$

$\Rightarrow \frac{9k+3}{k+1} = 5$

$\Rightarrow 9k+3 = 5k+5$

$\Rightarrow 4k = 2$

$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$

Hence point Q divides PR in the ratio $1:2$.

#418344

Topic: Section Formula

Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2, 4, 7)$ and $(3, -5, 8)$

Solution

Let the YZ plane divide the line segment joining points $(-2, 4, 7)$ and $(3, -5, 8)$ in the ratio $k:1$.

Using section formula, the coordinates of point of intersection are given by,

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the x-coordinate of any point is zero.

$\frac{3k-2}{k+1} = 0$

$\Rightarrow 3k - 2 = 0$

$\Rightarrow k = \frac{2}{3}$

Thus the YZ plane divides the line segment formed by joining the given points in the ratio $2:3$.

#418360

Topic: Section Formula

Using section formula show that the points $A(2, -3, 4)$, $B(-1, 2, 1)$ and $C\left(0, \frac{1}{3}, 2\right)$ are collinear

Solution

The given points are $A(2, -3, 4)$, $B(-1, 2, 1)$ and $C\left(0, \frac{1}{3}, 2\right)$

Let P be a point that divides AB in the ratio $k:1$.

Using section formula, the coordinates of P are given by,

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we will find the value of k at which point P coincides with point C.

$\Rightarrow \frac{-k+2}{k+1} = 0$, we get $k = 2$

For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$,

i.e., $C\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio $2:1$ and is the same as point P

Hence, points A, B and C are collinear.

#418361

Topic: Section Formula

Find the coordinates of the points which trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$

Solution

Given coordinates of P as $(4, 2, -6)$ and Q as $(10, -16, 6)$

Let R and S be the points of trisection of line segment PQ.

Then R divides PQ in the ratio $1:2$ and S is the mid-point of RQ.

Let the coordinates of R be (x, y, z)

By using section formula,

$x = \frac{1(10)+2(4)}{1+2}, y = \frac{1(-16)+2(2)}{1+2}, z = \frac{1(6)+2(-6)}{1+2}$

$\Rightarrow x = \frac{18}{3}, y = \frac{-12}{3}, z = \frac{-6}{3}$

$\Rightarrow x = 6, y = -4, z = -2$

So, the coordinates of R are $(6, -4, -2)$.

Now, since S is the mid-point of RQ.

#418413

Topic: Distance Between Two Points

Find the lengths of the medians of the triangle with vertices $A(0, 0, 6)$, $B(0, 4, 0)$ and $(6, 0, 0)$

Solution

Let AD, BE and CF be the medians of the given $\triangle ABC$.

Since AD is the median, D is the mid-point of BC.

$\therefore$ coordinates of point $D = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = (3, 2, 0)$

$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$

Since BE is the median, E is the mid-point of AC.

$\therefore$ coordinates of point $E = \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right) = (3, 0, 3)$

$BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$

Since CF is the median, F is the mid point of AB.

$\therefore$ coordinates of point $F = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0, 2, 3)$

Length of $CF = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

Thus the lengths of the medians of $\triangle ABC$ are 7, $\sqrt{34}$ and 7 units.

#418434

Topic: Section Formula

If the origin is the centroid of the triangle PQR with vertices $P(2a, 2, 6)$, $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find the values of a, b and c

Solution

The coordinates of the centroid of $\triangle PQR$

$= \left(\frac{2a-4-8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

It is given that origin is the centroid of $\triangle PQR$

$\therefore (0, 0, 0) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

$\Rightarrow \frac{2a+4}{3} = 0, \frac{3b+16}{3} = 0 \text{and} \frac{2c-4}{3} = 0$

$\Rightarrow a = -2, b = -\frac{16}{3}$ and $c = 2$

#418443

Topic: Distance Between Two Points

Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point $P(3, -2, 5)$

Solution

Let $A(0, b, 0)$ be the point on the y-axis at a distance of $5\sqrt{2}$ from point $P(3, -2, 5)$.

We have given, $AP = 5\sqrt{2}$

$\Rightarrow AP^2 = 50$

$\Rightarrow (3-0)^2 + (-2-B)^2 + (5-0)^2 = 50$

$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$

$\Rightarrow b^2 + 4b - 12 = 0$

$\Rightarrow b^2 + 6b - 2b - 12 = 0$

$\Rightarrow (b+6)(b-2) = 0$

$\Rightarrow b = -6, 2$

Thus, the coordinates of the required points are $(0, 2, 0)$ and $(0, -6, 0)$

#418449

Topic: Section Formula

A point R with x-coordinate 4 lies on the line segment joining the points $P(2,\ -3,4)$ and $Q(8,0,10)$. Find the coordinates of the point R.

Solution

The coordinates of points P and Q are given as $P(2,\ -3,4)$ and $(8,0,10)$

Let R divide line segment PQ in the ratio $k:1$

Hence by section formula, the coordinates of point R are given by,

$$\left(\frac{k(8)+2}{k+1},\frac{k(0)-3}{k+1},\frac{k(10)+4}{k+1}\right)=\left(\frac{8k+2}{k+1},\frac{-3}{k+1},\frac{10k+4}{k+1}\right)$$

It is given that the x-coordinate of point R is 4.

$\therefore \frac{8k+2}{k+1}=4$

$\Rightarrow 8k+2=4k+4$

$\Rightarrow 4k=2$

$\Rightarrow k=\frac{1}{2}$

Therefore, the coordinates of point R are $\left(4,\frac{-3}{\frac{1}{2}+1},\frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right)=(4,\ -2,6)$

#418457

Topic: Distance Between Two Points

If A and B be the points $(3,4,5)$ and $(-1,3,\ -7)$ respectively. Find the equation of the set of points P such that $PA^2\ +PB^2\ =K^2$, where K is a constant

Solution

Given coordinates of points A and B are $(3,4,5)$ and $(-1,3,\ -7)$ respectively.

Let the coordinates of point P be (x,y,z).

Given, $PA^2+PB^2=K^2$

$\Rightarrow (x-3)^2+(y-4)^2+(z-5)^2+(x+1)^2+(y-3)^2+(z+7)^2=K^2$

$x^2-6x+9+y^2-8y+16+z^2-10z+25+x^2+2x+1+y^2-6y+9+z^2+14z+49=K^2$

$\Rightarrow 2x^2+2y^2+2z^2-4x-14y+4z+109=K^2$

$\Rightarrow 2x^2+y^2+z^2-2x-7y+2z=K^2-109$

$\Rightarrow x^2+y^2+z^2-2x-7y+2z=\frac{K^2-109}{2}$

which is the required equation.

#418890

Topic: Introduction

Write the equations for the x-and y-axes.

Solution

The y-coordinate of every point on x-axis is 0.

Therefore, the equation of the x-axis is $y=0$.

And the x-coordinate of every point on y-axis is 0.

Therefore, the equation of the y-axis is $x=0$

#420631

Topic: Section Formula

In what ratio, the line joining $(-1,1)$ and $(5,7)$ is divided by the line $x+y=4$?

Solution

The equation of the line joining the points $(-1, 1)$ and $(5, 7)$ is given by,

$y - 1 = \frac{7-1}{5+1}(x+1)$

$y - 1 = \frac{6}{6}(x+1)$

$x - y + 2 = 0$......(1)

and the equation of the given line is

$x + y - 4 = 0$.....(2)

Thus point of intersection of lines (1) and (2) is given by,

$x = 1$ and $y = 3$

Let point $(1, 3)$ divide the line segment joining $(-1, 1)$ and $(5, 7)$ in the ration $1:k$.

Accordingly, by section formula,

$(1, 3) = \left\{\frac{k(-1) + 1(5)}{1+k}, \frac{k(1) + 1(7)}{1+k}\right\}$

$\Rightarrow (1, 3) = \left\{\frac{-k+5}{1+k}, \frac{k+7}{1+k}\right\}$

$\Rightarrow \frac{-k+5}{1+k} = 1, \frac{k+7}{1+k} = 3$

$\Rightarrow \frac{-k+5}{1+k} = 1$

$\Rightarrow -k + 5 = 1 + k$

$\Rightarrow 2k = 4 \Rightarrow k = 2$

Thus, the line joining the points $(-1, 1)$ and $(5, 7)$ is divided by line $x + y = 4$ in the ratio $1:2$.